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NEW SUPERSYMMETRY ALGEBRA ON GRAVITATIONAL INTERACTION OF NAMBU-GOLDSTONE FERMION

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Abstract

The spacetime symmetries of SGM action proposed as the gravitational coupling of N-G fermions are investigated. The commutators of new nonlinear supersymmetry (NL SUSY) transformations form a closed algebra, which reveals N-G fermion (NL SUSY) nature and a generalized general coordinate transformation. A generalized local Lorentz transformation, which forms a closed algebra, is also introduced.

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The supersymmetry (SUSY)[1] and its spontaneous breakedown are the essential notions to unify spacetime and matter. It is well understood that the Nambu-Goldstone (N-G) fermion with spin 1/2 would appear in the spontaneous breakdown of SUSY and that it can be converted to the longitudunal components of the spin 3/2 field (gravitino) through the superHiggs mechanism. This is demonstrated explicitly by the introduction of the local gauge coupling of Volkov-Akulov (V-A) model[2] of a nonlinear realization of SUSY (NL SUSY) to the supergravity (SUGRA) gauge multiplet[3].

In the previous papers [4] and [5], a supersymmetric composite unified model for spacetime and matter, superon-graviton model (SGM) based upon SO(10) super-Poincaré algebra, is proposed. The fundamental action which is the analogue of Einstein-Hilbert(E-H) action of general relativity(GR) describes the gravitational interaction of the spin 1/2 N-G fermions of V-A NL SUSY[2] regarded as the fundamental objects (superon-quintet) for matter. SGM may be the most economical model that accomodates all observed particles in a single irreducible representation of a (semi)simple group. The NL SUSY may give a framework to describe the unity of nature from the compositeness viewpoint for matter. In SGM all particles participating in (super)Higgs mechanism except graviton are composites of N-G fermions, superons. In ref.[5], we have constructed the gauge invariant SGM action based upon a new NL SUSY transformation and clarified the systematics in the unified model building.

We have further extended the framework [5] to N-G fermion with the higher spin in ref.[6] and obtained a new action(spin 3/2 SGM) analogous to E-H action of GR, where the action of spin 3/2 N-G fermion written down by Baaklini as a nonlinear realization of a superalgebra containing a vector-spinor generator[7] is extended systematically to the curved spacetime.

In this letter we demonstrate the (spacetime) symmetry of our SGM actions, especially we compute the commutators of new NL SUSY transformations on gravitational interaction of N-G fermion with spin 1/2 and 3/2 introduced in refs.[5] and [6]. We will show that the commutators of spacetime symmetries form a closed algebra, which reveals N-G (NL SUSY) nature of fermions and the invariances at least under a generalized general coordinate and a generalized local Lorentz transformations.

[I] The gravitational interaction of spin 1/2 N-G fermion:

We first summarize new NL SUSY transformations introduced in ref.[5]. In the arguments of SGM[5], we have regarded that the coset space (N-G fermion) SL(2,C) coordinates in addition to the SO(3,1) Lorentz coordinates are embedded at every curved spacetime point and introduce formally a new vierbein field ${w^a}_\mu(x)$ through

the NL SUSY invariant differential forms ω^a of V-A[2] as follows: †

$$\omega^a = w^a{}_\mu dx^\mu, \tag{1}$$
$$w^a{}_\mu(x) = e^a{}_\mu(x) + t^a{}_\mu(x), \tag{2}$$

where $e^a{}_\mu(x)$ is the vierbein of Einstein genaral relativity theory (EGRT) and $t^a{}_\mu(x)$ is defined by ‡

$$t^a{}_\mu(x) = ia\bar{\psi}\gamma^a\partial_\mu\psi \tag{3}$$

for spin 1/2 Majorana N-G field ψ. In Eq.(3) a is an arbitrary constant with the dimension of the fourth power of length (i.e., a fundamental volume of spacetime). We can easily obtain the inverse of the new vierbein, $w_a{}^\mu$, in the power series of $t^a{}_\mu$ which terminates with t^4:

$$w_a{}^\mu = e_a{}^\mu - t^\mu{}_a + t^\rho{}_a t^\mu{}_\rho - \cdots. \tag{4}$$

Similarly a new metric tensor $s^{\mu\nu}(x)$ is formally introduced in the abovementioned curved spacetime as follows:

$$s^{\mu\nu}(x) \equiv w_a{}^\mu(x) w^{a\nu}(x). \tag{5}$$

It is straightforward to show $w_a{}^\mu w_{b\mu} = \eta_{ab}$, $s_{\mu\nu} w_a{}^\mu w_b{}^\mu = \eta_{ab}$, ..etc.

In order to obtain simply the action in the abovementioned curved spacetime, which is invariant at least under GL(4,R), NL SUSY and local Lorentz transformations, we follow formally EGRT as performed in SGM[5]. That is, we require that the (mimic) vierbein $w^a{}_\mu(x)$ and the metric $s^{\mu\nu}(x)$ should have formally a general coordinate transformation under the supertranslations:

$$\delta x^\mu = -\xi^\mu, \quad \delta\psi = \zeta, \tag{6}$$

where $\xi^\mu = ia\bar{\zeta}\gamma^\mu\psi$ and ζ is a constant spinor parameter.
Remarkably the following nonlinear new (super)transformations

$$\delta\psi(x) = \zeta + ia(\bar{\zeta}\gamma^\mu\psi)\partial_\mu\psi, \tag{7}$$
$$\delta e^a{}_\mu(x) = ia(\bar{\zeta}\gamma^\rho\psi)D_{[\rho}e^a{}_{\mu]}, \tag{8}$$

induce the desirable transformations on $w^a{}_\mu(x)$ and $s^{\mu\nu}(x)$ as follows: §

$$\delta_\zeta w^a{}_\mu = \xi^\nu\partial_\nu w^a{}_\mu + \partial_\mu\xi^\nu w^a{}_\nu, \tag{9}$$
$$\delta_\zeta s_{\mu\nu} = \xi^\kappa\partial_\kappa s_{\mu\nu} + \partial_\mu\xi^\kappa s_{\kappa\nu} + \partial_\nu\xi^\kappa s_{\mu\kappa}. \tag{10}$$

† Latin $(a, b, ..)$ and Greek $(\mu, \nu, ..)$ are the indices for local Lorentz and general coordinates, respectively.

‡ In our convention $\frac{1}{2}\{\gamma^a, \gamma^b\} = \eta^{ab} = (+, -, -, -)$ and $\sigma^{ab} = \frac{i}{4}[\gamma^a, \gamma^b]$.

§ Throughout the paper D_μ is the covariant derivative of GL(4,R) with the symmetric affine connection.

That is, $w^a{}_\mu(x)$ and $s^{\mu\nu}(x)$ have general coordinate transformations under the new supertransformations (7) and (8).

In addition, to embed simply the local Lorentz invariance we follow EGRT formally and require that the new vierbein $w^a{}_\mu(x)$ should also have formally a local Lorentz transformation, i.e.,

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \tag{11}$$

with the local Lorentz transformation parameter $\epsilon_{ab}(x) = (1/2)\epsilon_{[ab]}(x)$.
Interestingly, we find that the following (generalized) local Lorentz transformations on ψ and $e^a{}_\mu$

$$\delta_L \psi(x) = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\psi, \tag{12}$$

$$\delta_L e^a{}_\mu(x) = \epsilon^a{}_b e^b{}_\mu + \frac{a}{4}\varepsilon^{abcd}\bar{\psi}\gamma_5\gamma_d\psi(\partial_\mu \epsilon_{bc}) \tag{13}$$

induce the desirable transformation (11). [Note that the equation (13) reduces to the familiar form of the Lorentz transformations if the global transformations are considered, e.g., $\delta_L g_{\mu\nu} = 0$.]

Therefore, replacing $e^a{}_\mu(x)$ in E-H Lagrangian of GR by the new vierbein $w^a{}_\mu(x)$ we obtain the analogue of E-H Lagrangian which is invariant under (7), (8), (12) and (13)[5]:

$$L = -\frac{c^3}{16\pi G}|w|(\Omega + \Lambda), \tag{14}$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu), \tag{15}$$

where the overall factor is now fixed uniquely to $\frac{-c^3}{16\pi G}$, $e_a{}^\mu(x)$ is the vierbein of EGRT and Λ is a probable cosmological constant. Ω is a mimic new scalar curvature analogous to the Ricci scalar curvature R of EGRT. The explicit expression of Ω is obtained by just replacing $e_a{}^\mu(x)$ in Ricci scalar R of EGRT by $w_a{}^\mu(x) = e_a{}^\mu + t_a{}^\mu$, which gives the gravitational interaction of $\psi(x)$. The lowest order term of a in the action (14) gives the E-H action of GR. And in the flat spacetime, i.e., $e_a{}^\mu(x) \to \delta_a{}^\mu$, it reduces to V-A model[2] with $\kappa^{-1} = \frac{c^3}{16\pi G}\Lambda$. Therefore our model(SGM) needs a non-zero (small) cosmological constant. Also from the low energy theorem viewpoint, the coupling constant of N-G fermion(superon) to the vacuum via the supercurrent is given $(\frac{c^3}{16\pi G}\Lambda)^{\frac{1}{2}}$.

The commutators of two new supersymmetry transformations (7) and (8) on $\psi(x)$ and $e_a{}^\mu(x)$ are calculated straightforwardly as [5]

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi = \{2ia(\bar{\zeta}_2\gamma^\mu\zeta_1) - \xi_1^\rho \xi_2^\sigma e_a{}^\mu(D_{[\rho}e^a{}_{\sigma]})\}\partial_\mu\psi, \tag{16}$$

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = \{2ia(\bar{\zeta}_2\gamma^\mu\zeta_1) - \xi_1^\sigma \xi_2^\lambda e_c{}^\rho(D_{[\sigma}e^c{}_{\lambda]})\}D_{[\rho}e^a{}_{\mu]} - \partial_\mu(\xi_1^\rho \xi_2^\sigma D_{[\rho}e^a{}_{\sigma]}). \tag{17}$$

These can be rewritten in the following familiar forms of the general coordinate transformations

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi = \Xi^\mu \partial_\mu \psi, \tag{18}$$

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = \Xi^\rho \partial_\rho e^a{}_\mu + e^a{}_\rho \partial_\mu \Xi^\rho, \tag{19}$$

where Ξ^μ is defined by

$$\Xi^\mu = 2ia(\bar{\zeta}_2 \gamma^\mu \zeta_1) - \xi_1^\rho \xi_2^\sigma e_a{}^\mu (D_{[\rho} e^a{}_{\sigma]}). \tag{20}$$

Therefore, the equations (7), (8), (18) and (19) reveal N-G fermion (NL SUSY) nature of $\psi(x)$, non-N-G nature of $e_a{}^\mu(x)$ corresponding to SGM scenario and generalized general coordinate transformations, which form a closed algebra.

Similarly, it is interesting to compute the commutator of the local Lorentz transformation on $e_a{}^\mu(x)$ of Eq.(13). It is calculated as

$$[\delta_{L_1}, \delta_{L_2}]e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{a}{4}\varepsilon^{abcd}\bar{\psi}\gamma_5\gamma_d\psi(\partial_\mu \beta_{bc}), \tag{21}$$

where $\beta_{ab} = -\beta_{ba}$ is defined by

$$\beta_{ab} = \epsilon_{2ac}\epsilon_1{}^c{}_b - \epsilon_{2bc}\epsilon_1{}^c{}_a. \tag{22}$$

Remarkably, the equations (13) and (21) explicitly reveal a generalized local Lorentz transformation with the parameters ϵ_{ab} and β_{ab} respectively, which shows the closure of the algebra. As for the internal symmetry, the global SO(N) symmetry can be introduced by replacing $\psi(x) \to \psi^i(x), (i = 1, 2, \ldots, N)$.
These arguments show that our action (14) with (3) is invariant at least under

$$[\text{global NL SUSY}] \otimes [\text{local GL}(4, \text{R})] \otimes [\text{local Lorentz}] \otimes [\text{global SO(N)}]. \tag{23}$$

SGM [5] for spacetime and matter is the case with N=10.

[II] The gravitational interaction of spin 3/2 N-G fermion:

The arguments are completely parallel with the spin 1/2 case. For the extension of the framework [5] to N-G fermion with spin 3/2[6], a new vierbein field $w^a{}_\mu(x)(= e^a{}_\mu(x) + t^a{}_\mu(x))$ is also introduced through the NL SUSY invariant differential forms ω_a of Baaklini[7] as (1) and (2) with

$$t^a{}_\mu(x) = ia\varepsilon^{abcd}\bar{\psi}_b\gamma_c\gamma_5\partial_\mu\psi_d \tag{24}$$

for spin 3/2 Majorana N-G field $\psi_a(x)$. As in the case for gravitational interaction of spin 1/2 N-G fermion, we require that the (mimic) vierbein $w^a{}_\mu(x)$ and the metric

$s^{\mu\nu}(x) \equiv w^{a\mu}(x) w_a{}^{\mu}(x)$ should have formally a general coordinate transformation under the supertranslations:

$$\delta x^\mu = -\xi^\mu, \quad \delta\psi^a = \zeta^a, \tag{25}$$

where $\xi^\mu = ia\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\nu\gamma_\rho\gamma_5\zeta_\sigma$ and ζ^a is a constant Majorana spinor parameter with spin 3/2. Remarkably we find that the following nonlinear new supertransformations

$$\delta\psi^a(x) = \zeta^a - ia(\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\nu\gamma_\rho\gamma_5\zeta_\sigma)\partial_\mu\psi^a, \tag{26}$$
$$\delta e^a{}_\mu(x) = -ia\varepsilon^{\rho\nu\sigma\lambda}\bar{\psi}_\nu\gamma_\sigma\gamma_5\zeta_\lambda D_{[\rho}e^a{}_{\mu]} \tag{27}$$

induce the desirable transformations on $w^a{}_\mu(x)$ and $s^{\mu\nu}(x)$ as (9) and (10). That is, $w^a{}_\mu(x)$ and $s^{\mu\nu}(x)$ have general coordinate transformations under the new supertransformations (26) and (27).

As for the Lorentz invariance we again require that the new vierbein $w^a{}_\mu(x)$ should have formally a local Lorentz transformation (11). Then we find that the following (generalized) local Lorentz transformations

$$\delta_L\psi^a(x) = \epsilon^a{}_b\psi^b - \frac{i}{2}\epsilon_{bc}\sigma^{bc}\psi^a, \tag{28}$$

$$\delta_L e^a{}_\mu(x) = \epsilon^a{}_b e^b{}_\mu - ia\varepsilon^{abcd}\{\bar{\psi}_b\gamma_c\gamma_5\psi_e(\partial_\mu\epsilon_d{}^e) - \frac{i}{4}\varepsilon_c{}^{efg}\bar{\psi}_b\gamma_g\psi_d(\partial_\mu\epsilon_{ef})\} \tag{29}$$

induce the desirable transformation (11). [The equation (29) also reduces to the familiar form of the Lorentz transformations if the global transformations are considered.]

Therefore, as in spin 1/2 SGM case, replacing $e^a{}_\mu(x)$ in E-H Lagrangian of GR by the new vierbein $w^a{}_\mu(x)$ defined by (2) with (24), we obtain the Lagrangian of the same form as (14), which is invariant under (26), (27), (28) and (29).

The commutators of two new supersymmetry transformations (26) and (27) on $\psi^a(x)$ and $e_a{}^\mu(x)$ are now calculated as[6]

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi^a = \{2ia(\varepsilon^{\mu bcd}\bar{\zeta}_{2b}\gamma_c\gamma_5\zeta_{1d}) - \xi_1^\rho\xi_2^\sigma e_a{}^\mu(D_{[\rho}e^a{}_{\sigma]})\}\partial_\mu\psi^a, \tag{30}$$

$$\begin{aligned}[\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = {} & \{2ia(\varepsilon^{\rho bcd}\bar{\zeta}_{2b}\gamma_c\gamma_5\zeta_{1d}) - \xi_1^\sigma\xi_2^\lambda e_c{}^\rho(D_{[\sigma}e^c{}_{\lambda]})\}D_{[\rho}e^a{}_{\mu]} \\ & -\partial_\mu(\xi_1^\rho\xi_2^\sigma D_{[\rho}e^a{}_{\sigma]}).\end{aligned} \tag{31}$$

These can be rewritten as

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi^a = \Xi^\mu\partial_\mu\psi^a, \tag{32}$$
$$[\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = \Xi^\rho\partial_\rho e^a{}_\mu + e^a{}_\rho\partial_\mu\Xi^\rho, \tag{33}$$

where Ξ^μ is now a generalized gauge parameter defined by

$$\Xi^\mu = 2ia(\varepsilon^{\mu bcd}\bar{\zeta}_{2b}\gamma_c\gamma_5\zeta_{1d}) - \xi_1^\rho\xi_2^\sigma e_a{}^\mu(D_{[\rho}e^a{}_{\sigma]}). \tag{34}$$

Therefore the equations (26), (27), (32) and (33) reveal N-G fermion (NL SUSY) nature of $\psi^a(x)$, non-N-G nature of ${e_a}^\mu(x)$ and a generalized general coordinate transformation, which form a closed algebra.

Also, the commutator of the local Lorentz transformation on ${e_a}^\mu(x)$ of Eq.(29) is calculated as

$$[\delta_{L_1}, \delta_{L_2}]{e^a}_\mu = {\beta^a}_b {e^b}_\mu - ia\varepsilon^{abcd}\{\bar\psi_b\gamma_c\gamma_5\psi_e(\partial_\mu {\beta_d}^e) - \frac{i}{4}{\varepsilon_c}^{efg}\bar\psi_b\gamma_g\psi_d(\partial_\mu\beta_{ef})\} \tag{35}$$

where β_{ab} is the same as (22). The equations (29) and (35) explicitly reveal a generalized local Lorentz transformation with the parameters ϵ_{ab} and β_{ab}, which forms a closed algebra.
Therefore our action (14) with (24), which is the analogue of the E-H action of GR, is invariant at least under

$$[\text{global NL SUSY}] \otimes [\text{local GL}(4, \text{R})] \otimes [\text{local Lorentz}] \otimes [\text{global SO(N)}], \tag{36}$$

when it is extended to global SO(N).

SGM formalism [5], i.e. the action (14) can be generalized to the spacetime with extra dimensions and to the inclusion of the non-abelian internal symmetries. It may give a potential new framework for the simple unification of spacetime and matter.

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